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Non-static spherically symmetric solution in a higher dimensional space time

Farook Rahaman*, Subenoy Chakraborty and Kallol Maity
Department of Mathematics, Jadavpur University, Kolkata-700 032, India
E-mail : jumath@cal.vsnl net.in

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Abstract : A nonstatic spherically symmetric solution in a higher dimensional space time is obtained. The energy momentum tensor for the model is taken as

$$T_t^t = T_r^r = \rho(r,t).$$

The exact solutions are obtained using functional separability of the metric coefficients.

Keywords : Spherical symmetry, higher dimension, general relativity.

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1. Introduction

In the early stages of evolution, it is expected that the Universe will be inhomogeneous to allow for generic initial conditions and formation of large scale structures in the Universe. For this reason, inhomogeneous (non-static) solutions of Einstein equations have been an interesting subjects for physicists [1]. Non-static solutions for spherically symmetric systems containing a perfect fluid of inhomogeneous matter density and pressure have been obtained in isotropic coordinates by several authors [2].

In last few years, there are attempts to unify gravity with other fundamental forces in nature. Latest studies of super string and super gravity theories and the Unification of fundamental forces with gravity reveal that the space time dimension should be different from four [3]. As a result, higher dimensional theory is receiving great attention both in cosmology and in particle physics. Solutions of Einstein field equations in higher dimensional space times are believed to be of physical relevance possibly at the extremely early times before the Universe underwent the compactifications transitions.

In this paper, we would like to consider a higher dimensional non-static spherically symmetric model with an adhoc energy momentum tensor.

We assume energy momentum tensors of the form

$$T_t^t = T_r^r = \rho, \quad T_\theta^\theta = T_\phi^\phi = T_\psi^\psi,$$

where ρ is an arbitrary function of r and t coordinates.

2. Basic equation

The metric for a five dimensional spherically symmetric, non-static space time can be taken in isotropic forms as

$$ds^2 = -A(r,t)dt^2 + B(r,t)dr^2 + r^2 H(t) \times \left[d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta \sin^2\phi d\psi^2\right]. \quad (1)$$

We take the energy-momentum tensors of the form

$$T_t^t = T_r^r = \rho(r,t); \quad T_\theta^\theta = T_\phi^\phi = T_\psi^\psi = 0. \quad (2)$$

The Einstein equations ($G_\mu^\gamma = 8\pi T_\mu^\gamma$) are

$$\frac{3}{4}\frac{\dot{B}\dot{H}}{ABH} + \frac{3}{4}\frac{\dot{H}^2}{AH^2} + \frac{3}{2}\frac{B'}{B^2 r} + \frac{}{r^2 H} \quad \frac{}{Br^2} = 8\pi\rho, \quad (3)$$

*Corresponding Author
Permanent Address : Khodar Bazar, Baruipur-743 303, 24 Parganas(S), West Bengal, India

$$\frac{3}{2}\frac{\ddot{H}}{AH}-\frac{3}{4}\frac{\dot{A}\dot{H}}{HA^2}-\frac{3}{2}\frac{A'}{rAB}+\frac{3}{r^2H}-\frac{3}{Br^2}=8\pi\rho, \quad (4)$$

$$\frac{\ddot{B}}{2AB}-\frac{\dot{B}^2}{4AB^2}+\frac{\ddot{H}}{AH}-\frac{\dot{H}^2}{4AH^2}+\frac{A'B'}{4AB^2}-\frac{A''}{2AB}$$
$$+\frac{A'^2}{4BA^2}-\frac{\dot{A}\dot{B}}{4BA^2}-\frac{\dot{A}\dot{H}}{2HA^2}-\frac{A'}{rAB}-\frac{1}{Br^2}$$
$$+\frac{\dot{B}\dot{H}}{2ABH}+\frac{B'}{rB^2}+\frac{1}{r^2H}=0, \quad (5)$$

and $$-\frac{\dot{H}}{rH}+\frac{\dot{B}}{rB}+\frac{1}{2}\cdot\frac{A'\dot{H}}{AH}=0. \quad (6)$$

Here, a dot and a prime denote partial differentiation with respect to t and r respectively.

The conservation equations ($T^{\alpha}_{\beta;\alpha}=0$) give the equations

$$\dot{\rho}+\rho\frac{\dot{H}}{H}=0 \text{ and } \rho'+\frac{2\rho}{r}=0. \quad (7)$$

These can be combined to give (after integration)

$$\rho=\frac{\rho_0}{r^2H}, \quad \rho_0=\text{constant}. \quad (8)$$

The general solutions for this space time are apparently quite difficult to obtain. We assume the metric coefficients to be separable in functions of r and t as

$$A=A_1(r)A_2(t);\; B=B_1(r)B_2(t). \quad (9)$$

3. Solution to the field equations

From (6) (using the separable form (9)), we get

$$B_2=B_0H^m;\; A_1=A_0r^{2(1-m)}, \quad (10)$$

where A_0, B_0 are constants of integration and m is the separation constant.

From the above field equations, we get

$$\frac{3}{4}\frac{\dot{B}_2\dot{H}}{A_2H}+\frac{3\dot{H}^2B_2}{4A_2H^2}-\frac{3}{2}\frac{\ddot{H}B_2}{A_2H}+\frac{3}{4}\frac{\dot{A}_2\dot{H}B_2}{HA_2^2}=-\alpha \quad (11)$$

and $$-\frac{3}{2}\frac{A_1'}{rB_1}-\frac{3}{2}\frac{B_1'A_1}{B_1r}=-\alpha. \quad (12)$$

Here, α is another separation constant.

Without any loss of generality, we can take $A_1=1$ (the value of A_2 different from unity only results a transformation of t coordinates)

Eqs. (12) and (13) take the following forms as

$$\dot{H}^2H^{m-2}(m+1)-2\ddot{H}H^{m-1}=-\frac{4}{3}\frac{\alpha}{B_0}, \quad (13)$$

$$\frac{B_1^1}{B_1^2}+\frac{1}{B_1}\frac{2(1-m)}{r}=\frac{2\alpha}{3A_0}r^{2m-1}. \quad (14)$$

Now after integrating, we get from (13)

$$\dot{H}^2=C_1H^{m+1}+\frac{4\alpha}{3B_0(1-2m)}H^{-m+2} \text{ (for } m\neq 1/2\text{)}, \quad (15)$$

(C_1 = constant of integration).

The integral form of (15) is

$$\pm(t-t_0)=\int\frac{dH}{\left[4\alpha/3B_0(1-2m)H^{-m+2}+C_1H^{m+1}\right]^{1/2}} \quad (16)$$

(for $m\neq\frac{1}{2}$)

We solve the eq. (15) and getting the expression of B_1 as follows :

$$B_1=\frac{1}{\left[C_2r^{2(1-m)}+\frac{\alpha}{3A_0(1-2m)}r^{2m}\right]} \quad (17)$$

(for $m\neq 1/2$).

(C_2 = constant of integration)

Case I : Let $C_1=C_2=0$ and $m<1/2$

In this case, we can easily solve the integral in (16) to give

$$H=t^{2/m} \quad (18)$$

Also from (17), we get

$$B_1=r^{-2m} \quad (19)$$

Let choose $T=\ln t$ and $R=\ln r$. Then the metric can be written as (except for a conformal factor)

$$ds^2=-dT^2+dR^2+e^{9mR+T(1/m-1)^2}d\Omega_3^2 \quad (20)$$

So we get a solid angle of deficit which depends both on radial and time coordinates.

Case II : Let $\alpha=0$

Then from (14), we get $H=t^{2/1-m}$,

Also from (15), we get $B_1=r^{2m-2}$.

Thus, we get a similar relation of the metric coefficients as in Case I.

Case III : $m=1$, $C_1\neq 0$, $C_2\neq 0$, $\alpha<0$. Take $\alpha=-\alpha'$ (where $\alpha'>0$)

The solution of the integral (16) as

$$H=\frac{2\alpha^1}{3B_0C_1}\sin h^2\frac{\sqrt{C_1}}{2}(t-t_0). \quad (21)$$

Also from (17), we get

$$B_1=\frac{1}{C_1+\alpha^1/3A_0r^2}. \quad (22)$$

Here, the explicit form of the metric (with a proper choice of the time coordinate and radial coordinate)

$$ds^2=H\left[-dT^2+dR^2+a^2\sin h^2Rd\Omega_3^2\right]. \quad (23)$$

Here, we take

$$r=a\sin h\, R,\; a=\text{constant and } T=\int\frac{dt}{\sqrt{H}}. \quad (24)$$

We see that the solution represents a static model and hence the solid deficit angle is a function of the radial coordinate only.

Case IV : $m = 0$, $C_1 \neq 0$, $C_2 \neq 0$, $\alpha < 0$. Also we take $\alpha = -\alpha'$ $(\alpha' > 0)$

Here, we solve the eq. (16) and getting an expression for H as

$$H = l\left[\sin\frac{\alpha'}{3B_0}(t-t_0)+\cos\frac{\alpha'}{3B_0}(t-t_0)\right]^2,$$

where

$$l^2 = \frac{C_1^2 B_0^2}{8^2\alpha'^2}. \qquad (25)$$

And the expression for B_1 as

$$B_1 = \frac{1}{C_2 r^2 + \alpha/3A_0}. \qquad (26)$$

Now, with a proper choice of radial coordinate, we get the explicit expression for the metric as

$$ds^2 = r^2\left(-dt^2 + dR^2 + H(t)d\Omega_3^2\right). \qquad (27)$$

This solution represents a time-dependent model and hence, we get a time-dependent solid deficit angle.

From the above result, it is evident that at any instant the energy density decreases with increasing r and vanishes at $r \to \pm\infty$.

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